



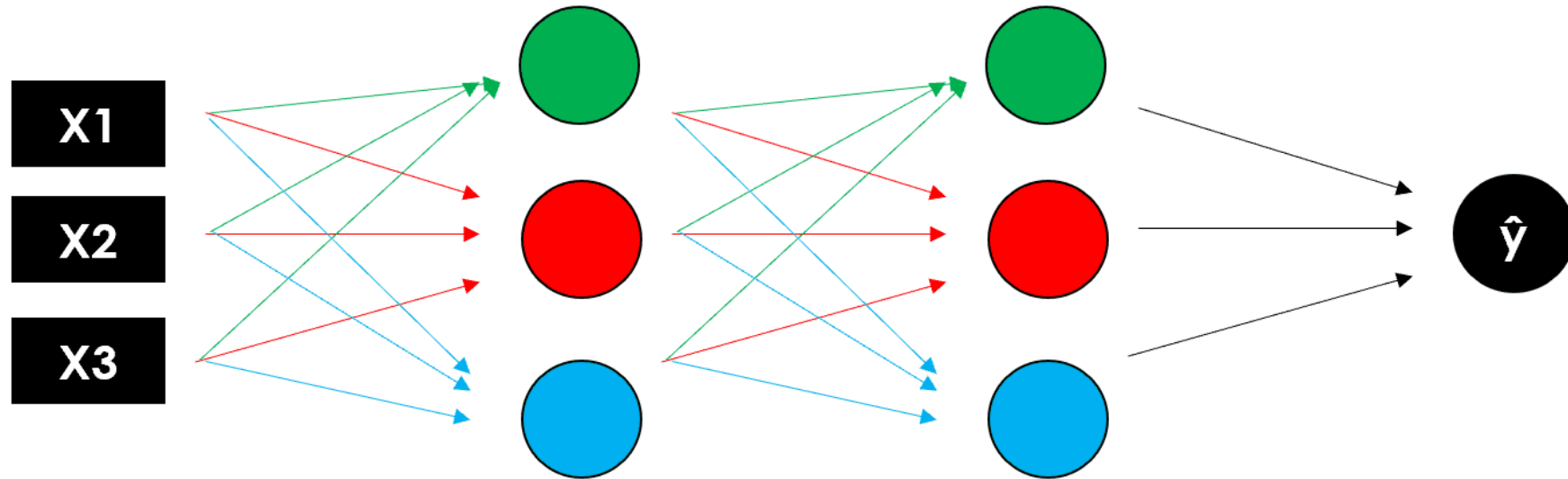
Introduction to Data Science

How Do We Represent and Compute Data?

8 Sep 2026

Why Linear Algebra for Deep Learning?

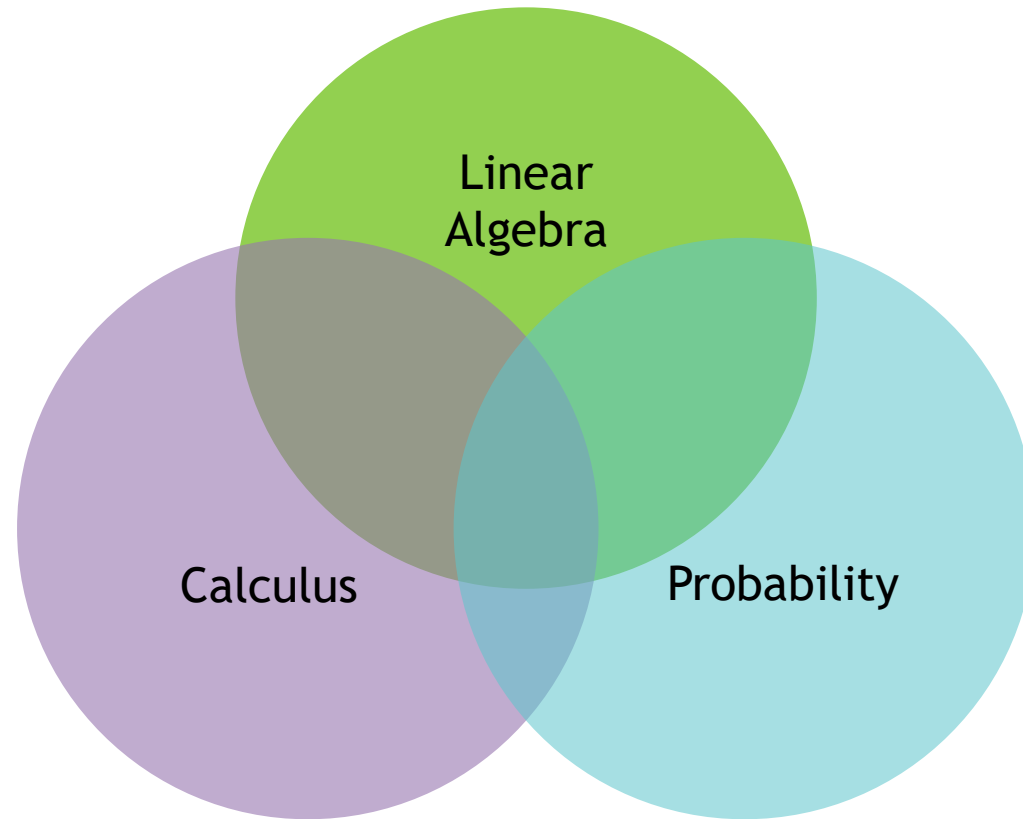
Fundamentals of Linear Algebra for Deep Learning



Deep Learning is built on mathematics.

Why Linear Algebra for Deep Learning?

Mathematics Behind Deep Learning



Today: Linear Algebra



Data Structures and Operations

Data Structures and Operations

What is Matrix

Matrix = A 2D array of Numbers

Columns →		0	1	2
Rows ↓	0	10	8	3
	1	5	20	6
	2	7	9	30

Columns: Vertical groups of values, indexed from left to right.

Rows: Horizontal groups of values, indexed from top to bottom

Data Structures and Operations

Example: Excel Spreadsheet

	Company X	Company Y
Salary	\$50,000	\$40,000
Bonus	\$5,000	\$7,500
Position	Engineer	Data Scientist

Question: Is this a matrix?

Data Structures and Operations

From Tables to Matrices

	Length	Weight	Speed	Age
Gecko Grayson	0.5m	0.1kg	4kph	10 years
Komodo Ken	2.5m	50kg	15kph	20 years
Iguana Ian	0.4m	9kg	30kph	15 years

Columns: Features ***Rows:*** Data points

Question: Why do we convert everything into numbers?

Data Structures and Operations

From Tables to Matrices

	Length	Weight	Speed	Age
Gecko Grayson	0.5m	0.1kg	4kph	10 years
Komodo Ken	2.5m	50kg	15kph	20 years
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→ $\begin{bmatrix} 0.5 & 0.1 & 4 & 10 \\ 2.5 & 50 & 15 & 20 \\ 0.4 & 9 & 30 & 15 \end{bmatrix}$

Columns: Features ***Rows:*** Data points

Answer: Efficient mathematical operations

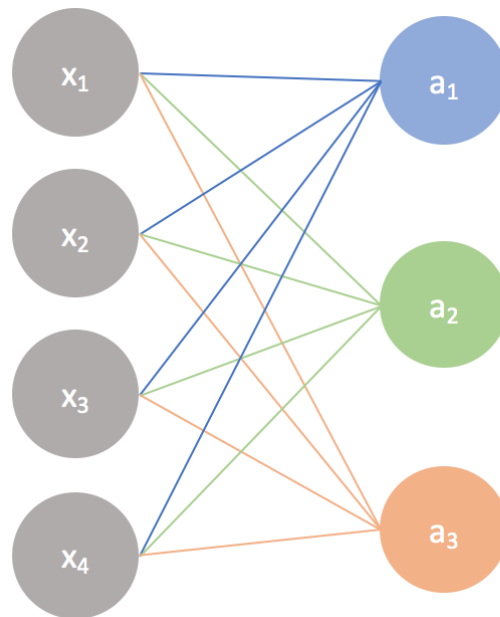
Data Structures and Operations

Why Numerical Representation?

Input layer

Output layer

Using multiple observations



$$\begin{bmatrix} w_1 & w_2 & w_3 & w_4 \\ w_1 & w_2 & w_3 & w_4 \\ w_1 & w_2 & w_3 & w_4 \end{bmatrix} \begin{bmatrix} \text{Observation 1} \\ \text{Observation 2} \\ \text{Observation 3} \\ \text{Observation 4} \end{bmatrix} + \begin{bmatrix} b \\ b \\ b \end{bmatrix} \xrightarrow{\text{activation}} \begin{bmatrix} \text{Observation 1} \\ \text{Observation 2} \\ \text{Observation 3} \\ \text{Observation 4} \end{bmatrix}$$

Efficient mathematical operations:

Image/Text $\rightarrow$ Numbers $\rightarrow$ Matrix/Tensor $\rightarrow$ Neural Network



Matrix Operations

Matrix Operations

Matrix Addition

$$\begin{bmatrix} 1 & 0 & 5 \\ 2 & 4 & 7 \end{bmatrix} + \begin{bmatrix} 5 & 3 & 2 \\ 6 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1+5 & 0+3 & 5+2 \\ 2+6 & 4+1 & 7+4 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 7 \\ 8 & 5 & 11 \end{bmatrix}$$

Same shape required

$$A - B = A + (-B)$$

Matrix Operations

Scalar-Matrix Multiplication

$$2 * \begin{bmatrix} 1 & 0 & 5 \\ 2 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 2 * 1 & 2 * 0 & 2 * 5 \\ 2 * 2 & 2 * 4 & 2 * 7 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 10 \\ 4 & 8 & 14 \end{bmatrix}$$

Multiply every element by a scalar

$$A - B = A + (-B)$$

Matrix Operations

Matrix Multiplication

$$A_{m \times k} B_{k \times n} = C_{m \times n}$$

$$\begin{array}{c}
 \text{A } (3 \times 2) \\
 \begin{array}{c} m = 3 \\ \text{(rows)} \end{array} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{array}{c} k = 2 \\ \text{(columns)} \end{array} \\
 \times \begin{array}{c} \text{B } (2 \times 4) \\ \begin{array}{c} k = 2 \\ \text{(rows)} \end{array} \begin{bmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{bmatrix} \begin{array}{c} n = 4 \\ \text{(columns)} \end{array} \\
 = \begin{array}{c} \text{C } (3 \times 4) \\ \begin{array}{c} m = 3 \\ \text{(rows)} \end{array} \begin{bmatrix} 29 & 32 & 35 & 38 \\ 73 & 80 & 87 & 94 \\ 117 & 128 & 139 & 150 \end{bmatrix} \begin{array}{c} n = 4 \\ \text{(columns)} \end{array}
 \end{array}$$

A is a 3×2 matrix
(3 rows, 2 columns)

B is a 2×4 matrix
(2 rows, 4 columns)

C is a 3×4 matrix
(3 rows, 4 columns)

Inner dimensions must match

Matrix Operations

Example: Matrix Multiplication

$$\begin{bmatrix} 4 & 6 \\ 2 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 9 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 4 * 1 + 6 * 5 & 4 * 9 + 6 * 2 \\ 2 * 1 + 7 * 5 & 2 * 9 + 7 * 2 \end{bmatrix} = \begin{bmatrix} 34 & 48 \\ 37 & 32 \end{bmatrix}$$

Row × Column → One number

Matrix Operations

Can We Reverse the Order?

$$(2 \times 3)(3 \times 5)$$

Can we compute BA ?

Matrix multiplication is not commutative.

$$AB \neq BA$$

$$(3 \times 5)(2 \times 3)$$

Matrix Operations

Matrix Multiplication: Another View

$$\begin{aligned} \begin{bmatrix} 4 & 6 \\ 2 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 9 \\ 5 & 2 \end{bmatrix} &= \begin{bmatrix} \begin{bmatrix} 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 5 \end{bmatrix} & \begin{bmatrix} 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 2 \end{bmatrix} \\ \begin{bmatrix} 2 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 5 \end{bmatrix} & \begin{bmatrix} 2 & 7 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 2 \end{bmatrix} \end{bmatrix} \\ &= \begin{bmatrix} 4 * 1 + 6 * 5 & 4 * 9 + 6 * 2 \\ 2 * 1 + 7 * 5 & 2 * 9 + 7 * 2 \end{bmatrix} \\ &= \begin{bmatrix} 34 & 48 \\ 37 & 32 \end{bmatrix} \end{aligned}$$

Matrix multiplication can also be viewed as a combination of column vectors.

Matrix Operations

Identity Matrix

$$AI = IA = A$$

What is the matrix version of multiplying by 1?

Matrix Operations

Matrix Inverse

$$\begin{bmatrix} \frac{-5}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{-1}{4} \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} \frac{-5}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{-1}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AA^{-1} = A^{-1}A = I$$

Matrix Operations

Why Do We Need an Inverse?

$$Ax = b$$

$$x = A^{-1}b$$

Inverse = Undoing a transformation